Z-Buffer Precision and Frustum Planes

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When a vertex is transformed the *scaled normal device coordinate* (SNDC) has the form (after homogenous division):

$$z_{NDC}^{S}(z) = S_1(a + \frac{b}{z}) + S_2$$

$$a = \frac{f+n}{f-n},\ b = \frac{2fn}{f-n}.$$

z_{NDC}^{S} is an *integer* value and stored in the z-buffer. For OpenGL the parameters *a* and *b* are computed from the frustum's near and far plane distance *n* and *f* in the above way. Because OpenGL uses the negative z-axis as viewing axis the near plane and the far plane intersect the camera's z-axis at –*n* and –*f*. The scaling parameters S_1 and S_2 are chosen that z_{NDC}^{S} ranges from 0 to $2^{zbufferbits}-1$, where *zbufferbits* is the number of bits used by the z-buffer.

We assume $f >> n$, so $a=1$ and $b=2n$.

To find a relation between the plane distances and the z-buffer precision we compute the change of the SNDC:

$$\Delta z_{NDC}^{S}(z) = \frac{\partial z_{NDC}^{S}}{\partial z}\Delta z = -\frac{2nS_1}{z^2}\Delta z = -\frac{2nS_1}{z}\frac{\Delta z}{z}$$

Now we ask how large the relative change of the z coordinate around the far plane's distance must be that the SNDC changes by 1 (remember that SNDC is an integer value). From the assumption

$$\Delta z_{NDC}^{S}(-f) = 1$$

follows

$$\frac{\Delta z}{z} = \frac{f}{2nS_1}.$$

We define the z-buffer precision as

$$\frac{\Delta z}{z} = F.$$

We observe that the value $2S_1$ is the range of the possible z-values (because the *normal device coordinates* range from -1 to 1):

$$2S_1 = 2^{zbufferbits}$$

From the last three equations we obtain a relation which tells us how far the near plane must be if the maximum z-buffer error (at the far plane) is *F*:

$$n > \frac{f}{F\,2^{zbufferbits}}.$$

Example: z-buffer has 16 bits, maximum error shall be 1% and far plane distance is 10000. For the near plane distance follows:

$$n > \frac{10000}{0{,}01 \cdot 65536} \approx 15{,}3$$

References:

- Steve Baker: *Learning to Love your Z-buffer*.
  `http://www.sjbaker.org/steve/omniv/love_your_z_buffer.html`